

INTEGRATING THE CONCEPTS OF DECISION RISK AND DECISION UNCERTAINTY WITH THE CONCEPTS OF DECISION TREES, BAYESIAN ANALYSIS, AND THE EXPECTED VALUE CRITERION

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ABSTRACT

It is argued that students' understanding of the concepts of decision risk and decision uncertainty can be improved substantially through exposure to the quantitative measurement of these concepts. This understanding is needed when assessing the value of a proposed marketing research study. Measures of both decision risk and decision uncertainty are described.

INTRODUCTION

If the amount of coverage devoted to a topic in marketing research textbooks is used as an indication of a topic's importance, assessing the value of research is an important topic. For example, Boyd, Westfall, and Stasch (1981), Churchill (1983), and Kinnear and Taylor (1983) devote 20 pages, 19 pages, and 10 pages respectively to this topic.

This topic is discussed in three different ways. Some books, such as Churchill's, stress the calculational process using decision trees, Bayesian analysis, and the expected value criterion. On the other hand, Aaker and Day (1983) identify and discuss the factors, such as decision uncertainty, which influence the value of research but barely discuss the calculational process. Still other books, such as Boyd, Westfall, and Stasch's, discuss both the factors that influence the value of research and the calculational process. However, none of the texts reviewed in preparing this paper integrate a conceptual discussion of two of the factors that influence the value of research into a discussion of the calculational process. These two factors are decision risk and decision uncertainty.¹ This general failure to integrate these concepts with the calculational process results in at least three specific problems. First, no text indicates that the value of perfect information is a function of the degree of risk involved in making a decision. Second, no text discusses how to measure decision risk. Without knowing how to measure risk, students might mistakenly conclude that risk can exist only when some payoffs are negative. Third, no text describes how to measure the degree of decision uncertainty. This omission makes it difficult for students to determine the degree of decision uncertainty and even difficult for them to fully understand the concept of decision uncertainty.

The main objective of this paper is to provide a more thorough understanding of the concepts of decision risk and uncertainty. This is accomplished by using concepts related to the calculational process to describe how to measure both decision uncertainty and risk.

¹In addition to these two factors, the perceived accuracy of the research results compared to the perceived accuracy of the prior probabilities affects the value of research. This factor is not discussed in this paper because textbooks generally have integrated this concept into the calculational process.

DEGREE OF DECISION UNCERTAINTY

Conceptually, a high degree of decision uncertainty exists if the decision is a "close call." Using decision tree terminology, a decision will be a close call when there is a small difference between expected values, using prior probabilities, of two or more decision alternatives. (The degree of uncertainty about states of nature also could be viewed as a determinant of decision uncertainty. However, the view adopted in this paper is that the effect of the degree of uncertainty about states of nature upon the value of research can be determined more conveniently when evaluating the confidence in the results of a proposed research study compared to the confidence in the prior probabilities than when examining decision uncertainty.)

The measure of decision uncertainty used in this paper is based upon the extent that the prior probabilities need to be revised to change the decision. This measure is directly related to the difference in expected values. As the difference between expected values decreases, the extent that prior probabilities need to be revised to change the decision decreases. In addition, this measure relates directly to the role that research can play in causing a revision of prior probabilities. With this measure, a unique algebraic solution exists for any pair of decision alternatives. This algebraic solution is first explained below in a situation in which only two alternatives and two states of nature exist. Then, the solution used when three or more alternatives and two states of nature exist is presented, followed by a discussion of the solution with two or more alternatives and three or more states of nature.

Two Alternatives and Two States of Nature

The basic idea in this situation is to find probabilities for each state of nature with which the expected value of the first alternative equals the expected value of the second alternative, with knowledge that the two probabilities must sum to 1.0. One of these probabilities is then subtracted from the corresponding prior probability. The degree to which the prior probabilities need to be revised to change the decision must be greater than this difference. This approach is presented in equation form below:

$$p_1^* + p_2^* = 1.0 \quad (1)$$

$$V_{11}p_1^* + V_{12}p_2^* = V_{21}p_1^* + V_{22}p_2^* \quad (2)$$

$$\text{Degree of Decision Uncertainty} > |p_1 - p_1^*| \quad (3)$$

An example is given below to demonstrate this procedure:

Payoff Table			
	S_1	S_2	$p(S_1) = .6$
A_1	100	-100	$p(S_2) = .4$
A_2	500	-500	
	$100 p_1^* - 100(1-p_1^*) = 500 p_1^* - 500(1-p_1^*)$		
	$p_1^* = .5$		
	Degree of Decision Uncertainty >	.6 - .5	
	Degree of Decision Uncertainty >	.1	

In this situation, A_2 would be selected using the expected value criterion and the prior probabilities. By increasing the probability of S_2 (where A_1 has a more favorable payoff) to .5 (a .1 change from .4) and decreasing the probability of S_1 (where A_1 has a less favorable payoff) to .5, the expected value of A_1 increases and the expected value of A_2 decreases just enough that the decision maker would be indifferent between A_1 and A_2 using the expected value criterion. Thus, any increase in the probability of $S_2 > .1$ would result in the decision maker changing the decision from A_2 to A_1 .

It should be noted that in the example given, A_2 has a more favorable payoff than A_1 if S_1 occurs while A_1 has a more favorable payoff if S_2 occurs. With the measure used, decision uncertainty can exist only if such a "tradeoff" exists. Also, it should be noted that a unique solution exists only because the probability of S_1 in combination with A_1 is the same as it is with A_2 . The validity of this assumption of the probability of a state of nature being independent of the decision alternative involved depends upon the particular states of nature selected. Thus, it is important to use this measure of decision uncertainty only when relevant states of nature can be selected whose probabilities are not influenced by the decision alternatives involved.

Three or More Decision Alternatives and Two States of Nature

The set of equations used in this situation is the same as used in the previous situation. Separate probabilities need to be calculated for each relevant pair of decision alternatives. To be relevant, each pair must include the alternative with the most favorable expected value based on the prior probabilities along with one of the other decision alternatives.

Two or More Decision Alternatives and Three or More States of Nature

In order to arrive at a unique algebraic solution in this situation, the probabilities of only two of the states of nature should be revised with each pair of decision alternatives. It is recommended that a systematic procedure be used to select these two states of nature with each pair of decision alternatives. The procedure used here, which is not the only procedure that could be used, involves selecting (1) the state of nature with which the first alternative has the greatest differential advantage, and (2) the state of nature with which the second alternative has the greatest differential advantage. This procedure results in the smallest possible revision in probabilities needed to achieve equal expected values with each decision alternative.

Equations (1) and (2) need to be modified slightly in this situation as shown below. Equation (3) should be used without modification.

$$p_i^* + p_j^* = p_i + p_j \quad (4)$$

$$\sum_{i=1}^2 v_{1i} p_i^* + \sum_{j=1}^n v_{1j} p_j = \sum_{i=1}^2 v_{2i} p_i^* + \sum_{j=1}^n v_{2j} p_j \quad (5)$$

The payoffs and prior probabilities below are used to demonstrate this procedure with one pair of decision alternatives.

	S_1	S_2	S_3	S_4	
A_1	500	300	-300	-500	$p(S_1) = .2$
A_2	-200	-100	100	200	$p(S_2) = .2$
					$p(S_3) = .4$
					$p(S_4) = .2$

$$p_1^* + p_4^* = p_1 + p_4 = .4$$

$$500 p_1^* - 500 (.4 - p_1^*) + 300(.2) - 300(.4) =$$

$$-200 p_1^* + 200 (.4 - p_1^*) - 100(.2) + 100(.4)$$

$$p_1^* = .257$$

$$\text{Degree of Uncertainty} > |.200 - .257| > .057$$

In this situation with S_1 and S_2 , A_1 has differential advantages of \$700 and \$400 respectively. With S_3 and S_4 , A_2 has differential advantages of \$400 and \$700 respectively. Thus, S_1 and S_4 are the states of nature selected for revision in the probabilities since A_1 has the greatest differential advantage with S_1 and A_2 has the greatest differential advantage with S_4 .

DECISION RISK

The particular decision risk involved when assessing the value of research is that of making a marketing mix decision without conducting research. This type of decision risk is a function of the consequences of selecting the marketing mix decision alternative that would be selected if the proposed research project is not conducted. Consequences will exist when a state of nature occurs with which the alternative selected without research does not have the most favorable payoff. The magnitude of the consequence, which also could be termed the opportunity loss of making the wrong decision, is equal to the difference between the most favorable payoff with a particular state of nature and the payoff, with the same state of nature, for the alternative that would be selected without the research.

Using the above view of decision risk, it should be measured by:

1. using prior probabilities to calculate expected values for each marketing mix decision alternative;
2. identifying the decision alternative which has the most favorable expected value; (This is the alternative that would be selected if the proposed research is not conducted.)
3. separately, for each state of nature, subtracting the payoff for the decision alternative identified in (2) above from the most favorable payoff for the state of nature; (This measures the magnitude of the consequence or opportunity loss of making a decision without research with each state of nature. Note that for those states of nature with which the alternative selected has the most favorable payoff, the difference will equal zero.)

4. weighting each difference by the relevant prior probability for each state of nature;
5. summing the weighted differences.

The payoffs and prior probabilities below are used to demonstrate this procedure:

	$S_1 = .6$	$S_2 = .3$	$S_3 = .1$
A_1	\$100	\$50	-\$50
A_2	\$50	\$100	-\$25
A_3	-\$50	\$0	\$80

- 1-2. A_1 has the most favorable expected value (\$70 compared to \$57.50 for A_2 and -\$22 for A_3)

$$\begin{array}{r}
 3. \qquad \qquad \qquad S_1 \\
 \frac{\$100 \text{ (most favorable payoff under } S_1)}{-\$100 \text{ (payoff for } A_1 \text{ under } S_1)}}{0} \\
 \qquad \qquad \qquad S_2 \\
 \frac{\$100 \text{ (most favorable payoff under } S_2)}{-\$50 \text{ (payoff for } A_1 \text{ under } S_2)}}{\$50} \\
 \qquad \qquad \qquad S_3 \\
 \frac{\$80 \text{ (most favorable payoff under } S_3)}{-(-\$50) \text{ (payoff for } A_1 \text{ under } S_3)}}{\$130}
 \end{array}$$

4-5. $\$0(.6) + \$50(.3) + \$130(.1) = \28

Although the above procedure is different than that used in most textbooks to determine the value of perfect information, the value calculated with this procedure always will equal the value of perfect information. Since the measured value of decision risk always equals the value of perfect information, both variables are measuring the same underlying concept. Thus, it makes little sense to talk about the two variables as separate concepts. In this author's opinion, the term "decision risk" more accurately describes the underlying concept than the term "value of perfect information." If the value of perfect information is a function of decision risk, it is appropriate to label the concept decision risk.

SUMMARY AND CONCLUSIONS

This paper has stressed the importance of discussing how to measure the concepts of decision uncertainty and decision risk as part of a more general discussion of the assessment of the value of research. None of the textbooks reviewed discuss the measurement of these two topics. Thus, unless or until these suggestions are incorporated into marketing research textbooks, the material presented in this paper needs to be discussed in the classroom. A recommended sequence and format to use in presenting this material in the classroom is provided below:

1. Describe the calculational process and basic terminology used (e.g., states of nature, expected value, conditional probabilities, Bayes Theorem). Students need to understand these concepts in order to grasp the ideas discussed in this paper.
2. After demonstrating the calculational process using a specific example, change the payoffs in such a way that the calculated value of research

increases because the degree of decision risk increases. Care should be taken to insure that the modification increases risk without, at the same time, increasing decision uncertainty. If the prior probabilities for each state of nature are equal, then interchanging payoffs for an alternative will increase risk without increasing uncertainty. For example, in the payoff table below, interchanging the payoffs of A_1S_2 and A_1S_3 will increase risk without increasing uncertainty.

	$S_1 = .333$	$S_2 = .333$	$S_3 = .333$
A_1	100	-50	50
A_2	50	-50	100

3. Ask students why the above change in payoffs increased the value of research. This gives students the opportunity to intuitively understand the concept of decision risk through self-insight
4. Discuss the manner in which decision risk should be measured.
5. Change the payoffs in an example of the calculational process in such a way that both the calculated value of research and decision uncertainty increase. Note that an increase in decision uncertainty will not always change the calculated value of research. It may not do so when a decision alternative, which has the most favorable payoff if a particular state of nature occurs, does not have the most favorable expected value, using the initial payoffs and conditional probabilities, even when the research result occurs that is most consistent with this state of nature. Using Churchill's example in Chapter two to demonstrate this idea, note that with the initial payoffs, the penetration pricing alternative (A_3) does not even have the most favorable expected value with research result Z_3 , which is the research result most consistent with the state of nature with which penetration pricing has the most favorable payoff (S_3). An increase in the payoff of A_3 when S_2 occurs from 0 to \$90 increases decision uncertainty but not the value of research.

Since many changes in payoffs which increase decision uncertainty also increase decision risk, care must be taken to change payoffs here so that decision uncertainty is increased without changing decision risk. Again using Churchill's example, changing the payoff of A_3S_1 from \$-50 to \$0 and changing the payoff of A_3S_2 from \$0 to \$90 would increase both decision uncertainty and the value of research without changing decision risk. However, changing the payoff of A_3S_3 from \$80 to \$180 would increase decision uncertainty and the value of research but also increase decision risk.

6. Ask students why the above change in payoffs increases the value of research. This gives students the opportunity to intuitively understand the concept of decision uncertainty through self-insight.
7. Discuss how decision uncertainty should be measured.

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